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Standardized Mortality Ratio and Confidence Interval

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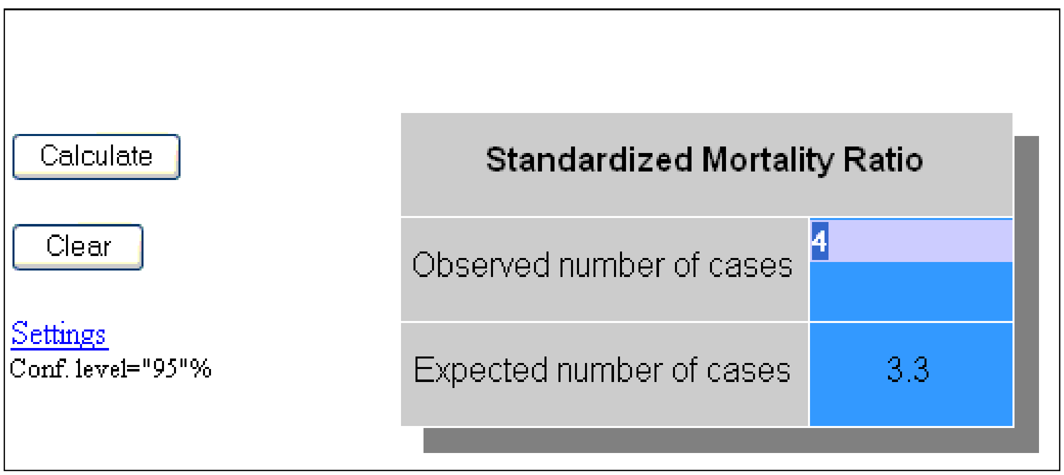
The Standardized Mortality Ratio (SMR) is the ratio of observed to the expected number of deaths in the study population under the assumption that the mortality rates for the study population are the same as those for the general population. For nonfatal conditions, the Standardized Mortality Ratio is sometimes known as the Standardized Morbidity Ratio.

This module tests for statistical significance and calculates various confidence intervals for SMR, based on a number of different methods. First, the user is prompted to enter observed and expected number of deaths in the respective data entry cells. Please note that the observed number of cases must be an integer as they are assumed to be Poisson variates (random variables with a Poisson distribution). The user can change the confidence interval settings as seen in the data entry dialog box (or) at 'Options/Setting' at the main menu screen.



The output from the example above is as follows:

SMR and 95% Confidence Interval

Observed number of cases	4
Expected number of cases	3.3

	Statistics	d.f.	p-value (2 sided)
Mid-P exact test	-	-	0.6571
Fisher exact test	-	-	0.8393
Byar approximation	0.206	-	0.8368
Chi-square test	0.1485	1	0.7

	Lower CL	SMR	Upper CL
Mid-P exact test	0.3851	1.212	2.924
Fisher exact test	0.3303		3.103
Byar approximation	0.3261		3.103
Rothman/Greenland method	0.455		3.229
Ury and Wiggins method	0.3274		3.036
Vandenbroucke method	0.3153		2.691

LookFirst items: Editor's choice of items to examine first.
Exact confidence intervals and p-values should be used when the number of observed deaths is less than or equal to five. For greater numbers of observed deaths, approximation methods are as nearly accurate as exact tests.
'?' = Not available.
Note..Only 90%, 95% & 99% confidence limits are available in Ury and Wiggins method, and Vandenbroucke method computes 95%CI only.

P-values are calculated under the assumption that the observed deaths are Poisson variates (random variables with a Poisson distribution) and the expected deaths are invariate. Exact confidence intervals and p-values should be used when the number of observed deaths is less than or equal to five. For greater numbers of observed deaths, approximation methods are as nearly accurate as exact tests.

In the output window, the statistical significance test between observed and expected number of deaths by Mid-P exact method shows p=0.6571.

The point estimate of SMR is 1.212, and six different methods are used to calculate the confidence interval around this estimate: Mid-P exact test, Fisher's exact test, normal approximation, Byar approximation, Rothman/Greenland method, Vandenbroucke method and

Ury & Wiggins method. Of these methods, the Mid-P exact test is generally the preferred method.

Based on p-values and confidence intervals that include null value ‘1’ in the output table, the interpretation is that there is no significant excess or deficit of mortality rate in the study population compared to that of general population.

For confidence limit estimates < 0.0, the value 0.0 is shown. All confidence intervals calculated are two-sided and depend on the setting of user’s choice (90%, 95%, 99%, 99.9% or 99.99%). Formulas for the methods are provided in the following section.

Formulae

The notation for the formulae is:

a = the observed number of deaths
λ = the expected number of deaths
SMR= a/b;
$Z_{1-\alpha/2}$ = the two-sided Z value

<u>**Significance Tests (two-tailed P-value)**</u>

Mid-P exact test (see Rothman and Boice):

If a>λ: $$p = \left(\frac{1}{2}\right)\frac{e^{-\lambda}\lambda^{a}}{a!} + \sum_{k=a+1}^{\infty}\frac{e^{-\lambda}\lambda^{k}}{k!}$$

If a<λ: $$p = \left(\frac{1}{2}\right)\frac{e^{-\lambda}\lambda^{a}}{a!} + \sum_{k=0}^{a-1}\frac{e^{-\lambda}\lambda^{k}}{k!}$$

Exact Test based on Poisson distribution (see Rosner):

If a<λ: $$p = \min\left[2 \times \sum_{k=0}^{a}\frac{e^{-\lambda}\lambda^{k}}{k!}, 1\right]$$

If a>λ: $$p = \min\left[2 \times \left(1 - \sum_{k=0}^{a-1}\frac{e^{-\lambda}\lambda^{k}}{k!}\right), 1\right]$$

Byar approximation (see Rothman and Boice):
If a>λ then a=a; If a<λ then a=a+1;

$$p = \sqrt{9 \times a} * \left[1 - \frac{1}{9 \times a} - \left(\frac{\lambda}{a} \right)^{\frac{1}{3}} \right]$$

<u>Calculation of Confidence Intervals</u>

Exact Tests (Mid-P and Fisher)
Exact confidence limits for an SMR can be derived by setting limits for the numerator and assuming the expected number in the denominator to be a constant. The limits for ‘a’ with 100(1-α) percent confidence are the iterative solutions $\underline{a}$ and $\overline{a}$.

Computing iterative solutions $\underline{a}$ and $\overline{a}$ is below……..

A. Mid-P exact test (see Rothman and Boice):

Lower bound: $\left(\frac{1}{2} \right) \frac{e^{-\underline{a}} \underline{a}^{a}}{a!} + \sum_{k=0}^{a-1} \frac{e^{-\underline{a}} \underline{a}^{k}}{k!} = 1 - \alpha / 2$

Upper bound: $\left(\frac{1}{2} \right) \frac{e^{-\overline{a}} \overline{a}^{a}}{a!} + \sum_{k=0}^{a-1} \frac{e^{-\overline{a}} \overline{a}^{k}}{k!} = \alpha / 2$

B. Fisher’s exact test (see Rothman and Boice):

Lower bound: $\sum_{k=0}^{a} \frac{e^{-\underline{a}} \underline{a}^{k}}{k!} = 1 - \alpha / 2$

Upper bound: $\sum_{k=0}^{a} \frac{e^{-\overline{a}} \overline{a}^{k}}{k!} = \alpha / 2$

Therefore, the exact lower and upper limits for SMR equal to "a/λ" would be

$\frac{\underline{a}}{\lambda}$ *and* $\frac{\overline{a}}{\lambda}$, respectively.

Byar Approximation: (see Rothman and Boice):

Lower bound: $$p = a\left[1 - \frac{1}{9a} - \frac{Z}{3}\sqrt{\frac{1}{a}}\right]^3$$

Upper bound: $$p = (a+1)\left[1 - \frac{1}{9(a+1)} + \frac{Z}{3}\sqrt{\frac{1}{a+1}}\right]^3$$

Rothman Greenland Method:

Lower bound: $$e^{\left[\ln(rate) - Z_{1-\alpha/2}\frac{1}{\sqrt{a}}\right]}$$

Upper bound: $$e^{\left[\ln(rate) + Z_{1-\alpha/2}\frac{1}{\sqrt{a}}\right]}$$

Ury & Wiggins Method: (only 90%, 95% and 99%CI available)

For 90%CI

Lower bound: $$SMR - Z_{1-\alpha/2}\sqrt{\frac{a}{\lambda}} + 0.65$$

Upper bound: $$SMR + Z_{1-\alpha/2}\sqrt{\frac{a}{\lambda}} + 1.65$$

For 95%CI

Lower bound: $$SMR - Z_{1-\alpha/2}\sqrt{\frac{a}{\lambda}} + 1$$

Upper bound: $$SMR + Z_{1-\alpha/2}\sqrt{\frac{a}{\lambda}} + 2$$

For 99%CI

Lower bound: $SMR - Z_{1-\alpha/2}\sqrt{\frac{a}{\lambda}+2}$

Upper bound: $SMR + Z_{1-\alpha/2}\sqrt{\frac{a}{\lambda}+3}$

Vandenbroucke Method: (only 95%CI available)

$$\frac{\left(\sqrt{a} \pm Z_{1-\alpha/2} \times 0.5\right)^2}{\lambda}$$

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