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Confidence Intervals for a Rate

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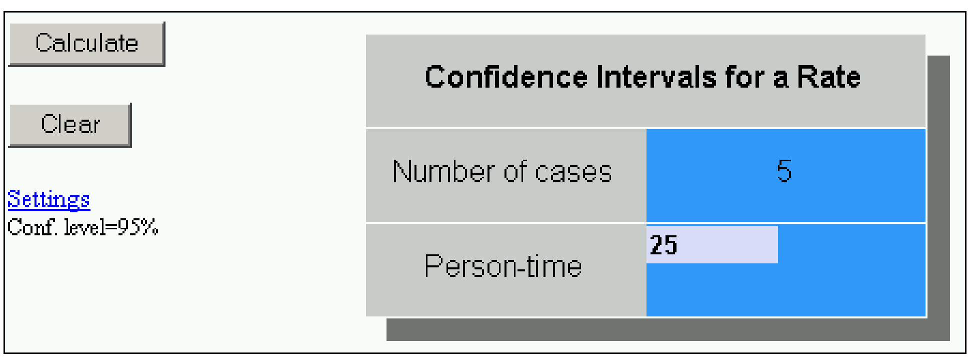
The Person Time module of Open Epi is used to analyze data where the numerator is a count of the events of interest and the denominator is the total person-time over which observations occurred. This method of analysis is frequently used in cohort studies and clinical trials. The idea is that a disease-free population is followed from a baseline. Person-time is the amount of time an individual accumulates until: 1) the study ends; 2) they develop the outcome of interest; or 3) they leave the study for some other reason. Person time is frequently expressed in person-years, although person-hours, days, or months will work just as well.

Single Person-Time Rate

For a single rate (also known as “incidence rate”), the numerator is the number of cases of the “disease,” and the denominator is the sum of person-years (or days, weeks, months) of exposure for all individuals prior to onset of the disease. The person-time variable represents the sum of the number of time units in which individuals were under study and disease-free. It should include units for those who never developed disease and those who were lost to follow-up after a defined period.

This module calculates various confidence intervals for a rate. First, the user is prompted to enter a numerator and denominator value:



The output from the example above is as follows:

Case-Count and Person-Time Data

Number of cases:	5
Person-Time:	25

Person-Time Rate and 95% Confidence Intervals

Per 100 Person-Time Units

	Lower CL	Rate	Upper CL
Normal approximation	2.471	20	37.53
Byar approx. Poisson	6.446		46.67
Rothman/Greenland	8.325		48.05

"Normal approximation" to the Poisson distribution as described by Rosner, Fundamentals of Biostatistics (5th Ed)
"Byar approximation to the Poisson" as described in Rothman and Boice, Epidemiologic Analysis with a Programmable Calculator
"Rothman/Greenland" as described in Rothman and Greenland, Modern Epidemiology (2nd Ed)

The observed rate is 20 per 100 person-time units. Three different methods are used to calculate the confidence interval around this point estimate: the normal approximation, Byar approximation, and the Rothman/Greenland method. Of the three methods, the Rothman/Greenland is generally the preferred method.

For confidence limit estimates < 0.0, the value 0.0 is shown. Currently all confidence intervals calculated are two-sided 95% confidence intervals. Formulas for the methods are provided in the following section.

Formulae

The notation for the formulae is:

a = the observed numerator
PT = is the observed denominator in person-time units
$rate = a/PT$
$Z_{1-\alpha/2}$ = the two-sided Z value, 1.96 for a 95% confidence interval

Normal Approximation:

$$rate \pm Z_{1-\alpha/2}\sqrt{\frac{a}{PT^2}}$$

Byar Method (see Rothman and Boice):

Lower bound: $a\left(1-\dfrac{1}{9a}-\dfrac{Z_{1-\alpha/2}}{3}\sqrt{\dfrac{1}{a}}\right)^3$

Upper bound: $(a+1)\left(1-\dfrac{1}{9(a+1)}+\dfrac{Z_{1-\alpha/2}}{3}\sqrt{\dfrac{1}{a+1}}\right)^3$

Rothman Greenland Method:

Lower bound: $e^{\left[\ln(rate)-Z_{1-\alpha/2}\frac{1}{\sqrt{a}}\right]}$

Upper bound: $e^{\left[\ln(rate)+Z_{1-\alpha/2}\frac{1}{\sqrt{a}}\right]}$

References

Rosner B. Fundamentals of Biostatistics, 5th Edition. Duxbury Press, 2000.

Rothman KJ, Boice JD Jr: Epidemiologic analysis with a programmable calculator. NIH Pub No. 79-1649. Bethesda, MD: National Institutes of Health, 1979;31-32.

Rothman KJ, Greenland S. Modern Epidemiology, 2nd Edition. Lippincott-Raven Publishers, Philadelphia, 1998.